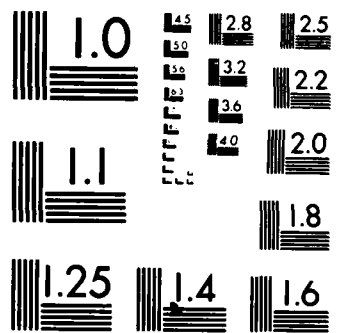


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ELECTRICAL AND COMPUTER ENGINEER. Y LIU ET AL.  
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**AFOSR-TR- 86 - 0795**  
**AN ANALYSIS OF A CSMA/CD COLLISION RESOLUTION SCHEME**

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### ABSTRACT

In this paper we analyze the performance characteristics of a packet broadcasting random multiple access computer communication network with a CSMA/CD protocol. The analysis is based on the Enet II protocol, which was designed to effectively resolve collisions in such a network. We establish bounds on the performance of the network.

### I. INTRODUCTION

In the past few years, packet broadcasting random multiple access computer communication networks have become commercially available. A typical example of such a network is the Ethernet developed by Xerox, which was designed based on the idea of carrier sense multiple access with collision detection (CSMA/CD) [1]. The basic Ethernet protocol is described in the IEEE standard 802.3, where a station among a number of users sharing a common broadcast channel will listen before transmitting, and defer if the channel is busy. In Ethernet, when two or more stations collide, a retransmission is scheduled for a later time. Each colliding station waits for a random period of time before retransmitting. The mean of this randomized waiting period before retransmission is increased under times of heavy load in order to sustain channel efficiency. Although Ethernet has the advantage of easy interconnection of stations to the common channel and it provides a high level of utilization of the channel [2], it does not truly address the problem of how to effectively resolve collisions in the channel. Thus packets involved in a collision may incur excessive delay due to waiting and abortion of transmission. This is even less desirable if a large number of stations are connected to the network and channel access demand is high.

### II. PRELIMINARIES

Recently, a protocol called Enet II was introduced by Molloy [3] as a candidate for the second generation of Ethernet. This protocol is designed to effectively resolve contention in a broadcast multiple access network such as Ethernet. Before we describe the Enet II protocol, we will introduce some notation. We will assume that the "diameter" of the network is given, that is, the maximum propagation delay between any two stations in the network. Let  $r$  be twice the diameter of the network. Any station listening to the channel for an interval of  $r$  units of time after transmitting would be guaranteed to hear something if anyone else were attempting to use the channel during that interval of time. A collision occurs when two or more stations attempt to transmit within an interval of  $r/2$  units of time. According to the protocol, the stations are in one of three states: inactive, active, and deferred. Inactive stations either do not have anything to send or have just finished sending something. Active stations

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are trying to send a packet (which might be a new message or might be a message involved in a previous collision). Deferred stations have attempted to transmit but are waiting for the active stations to leave the active state. We also assume that each station has a coin flipping mechanism such that the probability of head appearing is  $p$ . The Enet II protocol is given by the following procedure.

Inactive stations (with a packet generated):

Follow normal CSMA procedure (i.e., check channel before going to active state).

If channel is idle, wait for  $3r$  units of time; then transmit.

If channel is busy, wait until it is idle for  $3r$  units of time; then transmit, and the station goes to the active state.

Active stations:

If transmission is successful, station returns to the inactive state.

If a collision occurs, all participants in that collision flip a coin with probability of head equalling  $p$ .

If "head" appears, the station tries to transmit again.

If "tail" appears, the station monitors the channel passively:  
if the station sees the channel idle for  $r$  units of time, transmit;

if the station sees a successful transmission, wait for the end of that transmission and then transmit; and

if the station sees a collision, the station changes to the deferred state.

Deferred stations:

Passively monitor the channel.

If the station sees the channel idle for an interval of  $2r$  units of time, it transmits and returns to the active state.

If the station sees the channel as not idle in an interval of  $2r$  units of time, it remains in the deferred state.

The Enet II protocol is simple and needs no extensive support, such as clocks, addresses, current load estimates, or preassigned orderings, as compared with some other contention resolution protocols [4,5]. This protocol is characterized by the introduction of a "gate" for new arrivals such that stations have to wait for the channel to be idle for a period of  $3r$  units of time before transmitting a new packet. Therefore, stations need not monitor the channel when they have nothing to send. All new arrivals must stay behind that gate until all active or deferred users, if any, are finished. Similarly, the deferred users must stay behind their gate for a period of  $2r$  units of time until the active users are finished. Assuming at least one success and at least one failure among the Bernoulli trials generated by the active users, the random test mechanism will decrease the number of active users participating in a collision by successfully transmitting some or having them move to a deferred state in the case that it is known that two or more stations are still in the active state. Active stations which flipped "tail" in their coin flipping tests still transmit after the channel is free for  $r$  units of time, effectively announcing their presence to keep deferred stations from erroneously concluding that all active stations are done. When all of the active stations transmit successfully, all of the deferred stations will change to the active state.

### III. DEVELOPMENT

In [6], we performed a preliminary analysis of the Enet II protocol by assuming a simplified model of  $n$  stations in the network, where each station generates packets of equal length. In this section, we will investigate



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the performance of the Enet II protocol while stations in the network generate packets of variable packet lengths.

We assume that there are  $n$  stations in the network, and we index these stations from 1 to  $n$ . Assume that each of these  $n$  stations is equally likely to have a packet ready to send out and that the packet arrivals at the different stations are mutually independent. In the context of no collision, let  $\tau_i$  denote the time from when the  $i$ -th station transmits a packet until the time that packet exits the channel. When the  $i$ -th station involved in a collision successfully transmits its packet, let  $\tau_i$  denote the time from when the packet is last transmitted by the  $i$ -th station until the time that packet exits the channel. We will call the  $\tau_i$ 's the packet transmission times, and we will model  $\tau_i$ ,  $1 \leq i \leq n$  as a random variable.

Note that this does not exclude the possibility of them being constants. Assume that the  $\tau_i$ 's are mutually independent and that the minimum of the support of each random variable  $\tau_i$  is always greater than  $r/4$ . The latter assumption guarantees that, in case of a collision, the time needed to witness the collision is less than the time needed to resolve the collision. (In local area networks, the usual maximum spatial separation among the stations is less than 10 Km, and the assumption that  $\tau_i > r/4$  is usually easily satisfied.)

A station having a packet ready to send listens before transmitting. As we have noted, if more than two stations try to transmit their messages within a period of  $r/2$  units of time, a collision is said to have occurred. A collision will be detected and transmission of all colliding stations will be aborted. Then the stations will flip coins and retransmit according to the outcome of the coins. Let the random variable  $Z$  denote the time between when the first packet was sent in a collision and the time when the colliding stations acknowledge the collision and flip coins. We assume that the mean of  $Z$  is independent of the number of stations involved in a collision. Let  $\delta = E[Z]$ .

In the context of the resolution of a collision, assume that there are  $j$  ( $j > 1$ ) stations monitoring a transmission and waiting for it to end and for the channel to be free. We will let  $\mu_j$  be the average time until the first of these  $j$  waiting stations witnesses the end of the current transmission. In this context we use the word average in the sense that the average is over all possible combinations of  $j$  waiting stations and one transmitting station. For each combination there is an associated time for the first to witness the end of the current successful transmission. There is also a probability associated with each combination. This probability is obviously dependent upon the packet arrival statistics, and it is also dependent on the coin flipping outcomes. We note that  $\mu_j \leq r/2$ ,  $1 \leq j \leq n$ . We will let  $\mu_0 = 0$ .

Consider the successful transmission of a packet. It is transmitted either without having experienced a collision or during the resolution of a collision. In the absence of a collision, the average time required for a successful transmission will be denoted by  $C_1$ , and is given by

$$C_1 = \frac{1}{n} \sum_{i=1}^n E[\tau_i],$$

since each of the stations was assumed to be equally likely to have a packet ready to send out. If the packet is transmitted during the resolution of a collision, then the transmission occurred when the transmitting station flipped "head" and each of the others either flipped "tail" or was

in the deferred state. In a collision, each station is as equally likely to be involved, and due to the coin flipping mechanism each station in the collision is as equally likely to be the one transmitting during a successful transmission. Thus  $C_1$  given above is the average time to execute a

successful transmission either during a collision or in the absence of one.

In [6], it is assumed that all packets transmitted in the channel are of the same length, and it takes  $\tau$  units of time to transmit a packet without experiencing a collision. The results in [6] are therefore a special case of the following analysis by taking  $\tau_1 = \dots = \tau_n = \tau$ , and hence  $C_1 = \tau$ .

Consider a  $k$ -way collision,  $2 \leq k \leq n$ , and let  $C_k$  be the average time between when the first packet was sent but ends up in a  $k$ -way collision and when the very last packet in this collision is successfully transmitted. The average is taken over all possible choices of  $k$  colliding stations of  $n$  stations in the network. Then by using the law of total probability,  $C_k$  is given by

$$C_k = C(k, 0, 0) \quad \text{for } k \geq 2,$$

where the  $C(i, j, k)$ 's satisfy the following difference equations

$$C(0, 0, 0) = 0$$

$$C(0, j, k) = r + C(j, 0, k), \quad \text{for } j \geq 1, \quad (1)$$

$$C(0, 0, k) = \mu_k + 2r + C(k, 0, 0), \quad \text{for } k \geq 1, \quad (2)$$

$$C(1, j, k) = C_1 + \mu_j + C(j, 0, k), \quad (3)$$

where, for  $i \geq 2$ ,

$$C(i, 0, k) = [\delta + \sum_{\ell=1}^{i-1} \binom{i}{\ell} p^\ell (1-p)^{i-\ell} C(\ell, i-\ell, k) + r(1-p)^i] / [1-p^i - (1-p)^i], \quad (4)$$

and

$$C(i, j, k) = \delta + \sum_{\ell=1}^{i-1} \binom{i}{\ell} p^\ell (1-p)^{i-\ell} C(\ell, i-\ell, j+k) + [p^i + (1-p)^i] C(i, 0, j+k) + r(1-p)^i. \quad (5)$$

The arguments of  $C(i, j, k)$  can be interpreted as  $i$  corresponding to the number of active stations ready to transmit their messages,  $j$  corresponding to the number of active stations flipping "tail" after a collision and passively monitoring the channel, and  $k$  corresponding to the number of stations in the deferred state; and  $C(i, j, k)$  can be interpreted as the average time between when the situation corresponding to the arguments first occurs in executing the Enet II protocol until the last packet in this situation is successfully transmitted. In the context of (5), two or more stations attempt to transmit (i.e.  $i \geq 2$ ); it takes an average of  $\delta$  units of time to detect the collision, and the remaining terms in (5) represent outcomes of coins flipped by the contending  $i$  stations and the corresponding average times to resolve the situations associated with these outcomes. Equation (4) is a special case of (5) obtained by setting the second argument to zero. The boundary conditions given by (1)-(3) are obtained according to the protocol. Equation (1) represents the case where  $j$  stations flipping "tail" see the channel being free for  $r$  units of time and try to gain access to the channel; the  $k$  deferred stations remain in the deferred state in this case. Equation (2) represents the case where upon sensing the completion of a successful transmission and the ensuing idleness of the channel, the  $k$  stations wait  $2r$  units of time and then attempt to access the channel; recall that  $\mu_k$  is the average time until the first of  $k$  stations witnesses the completion of the successful transmission. Equation (3) corresponds to the case where one station is transmitting; it

takes an average of  $C_1$  units of time to finish this single transmission and then it takes an average of  $\mu_j$  units of time until the first of the  $j$  stations flipping "tail" witnesses the end of the successful transmission; these  $j$  stations attempt to access the channel and the  $k$  deferred stations remain in the deferred state.

From (1)-(5) we can obtain  $C_k$ ,  $2 \leq k \leq n$ . For example,  $C_2 = \mu_1 + 2C_1 + [\delta + r(1-p)^2]/[2p(1-p)]$ . For  $k \geq 3$ ,  $C_k$  can be calculated recursively by aid of a computer. From the recursiveness of (1)-(5), we observe that  $C_k$ ,  $2 \leq k \leq n$ , is a positive function of  $p$  in the open interval  $(0,1)$ . Also,  $C_k$  is  $+\infty$  if  $p$  is equal to 0 or 1, since in either case a  $k$ -way collision can never be resolved. By a limiting argument, we can see that  $C_k$  approaches  $+\infty$  as  $p$  approaches 0 and 1. Thus  $C_k$  has a minimum and can be minimized by choice of  $p$ . Note also that  $C_k$  is always bounded by  $kC_1$ , which is the average time to transmit  $k$  packets sequentially and successfully. We will call  $C_k - kC_1$  the average collision resolution time since this is the average of the extra time not accounted for in the actual transmission but rather in resolving the collision. Since  $C_2 - 2C_1$  is not dependent upon  $C_1$ , it follows from the recursive nature and an induction argument that the average collision resolution time  $C_k - kC_1$ ,  $2 \leq k \leq n$ , is independent of  $C_1$ . Hence  $C_k$  is a sum of two terms: the average overall time to transmit  $k$  packets sequentially, and the average collision resolution time. In Fig. 1 we present a plot of the average collision resolution time for various values of  $k$ . In this plot, we assume that  $C_1$ , the average time to transmit a randomly chosen packet, is fixed. Note that this plot for average collision resolution time is independent of  $C_1$ . We also observe that the minimizing  $p$  for each  $C_k - kC_1$  is different and is not equal to  $1/2$  for any  $k$ .

Another factor on which  $C_k$  depends is  $\delta$ , the average time from when the first colliding packet is sent until the coins are flipped. Obviously  $\delta$  depends upon the characteristics of the individual facilities in practice. In Fig. 2, we present a plot of  $C_4 - 4C_1$  versus  $p$  for various values of  $\delta$ . We observe that for larger  $\delta$ , the average collision resolution time  $C_4 - 4C_1$  is longer. We also note that it follows from the preceding recursive equations that for a fixed  $p$ ,  $C_k$  is an affine function of  $\delta$  for  $k \geq 2$ .

We note that in (2) and (3),  $\mu_j$ ,  $1 \leq j \leq n$ , is upper bounded by  $r/2$ . Due to the recursive nature of (1)-(5), we can upper bound  $C_k$ ,  $2 \leq k \leq n$ , by  $C_k^*$ , which is obtained through (1)-(5) by setting  $\mu_j$  and  $\mu_k$  each equal to  $r/2$  in (2) and (3), respectively.

In the context of a  $k$ -way collision ( $2 \leq k \leq n$ ), we will now consider the situation where a particular station involved in the collision, say station  $m$ , is concerned with how long it will take to successfully transmit its packet. Let  $L_k$  be the average time from when the first packet involved in a  $k$ -way collision was transmitted until when the packet of interest is successfully sent. This average is again taken over all possible choices of  $k$  colliding stations including the station of interest among all  $n$  stations in the network. Then by the use of the law of total probability and a similar argument used in obtaining the  $C_k$ 's,  $L_k$ ,  $2 \leq k \leq n$  is given by



$L_k = L(\underline{k}, 0, 0)$  where  $L(\underline{k}, 0, 0)$  satisfies the following recursive equations:

$$L(0, 0, 0) = 0,$$

$$L(0, 0, k) = \mu_k + 2r + L(\underline{k}, 0, 0), \quad \text{for } k \geq 1,$$

$$L(\underline{1}, j, k) = E[\tau_m]$$

$$L(\underline{1}, j, k) = C_1 + \mu_j + L(\underline{j}, 0, k), \quad \text{for } j \geq 1,$$

$$L(\underline{1}, j, k) = C_1 + \mu_j + L(j, 0, \underline{k}), \quad \text{for } k \geq 1,$$

where for  $i \geq 2$ ,

$$L(\underline{i}, j, k) = \delta + L(\underline{i}, 0, j+k),$$

$$L(i, \underline{j}, k) = \delta + L(i, 0, \underline{j+k}),$$

$$L(\underline{i}, 0, k) = \left\{ \delta + \sum_{\ell=1}^{i-1} p^\ell (1-p)^{i-\ell} \left[ \binom{i-1}{\ell-1} L(\underline{\ell}, i-\ell, k) \right. \right. \\ \left. \left. + \binom{i-1}{\ell} L(\ell, i-\ell, k) \right] + r(1-p)^i \right\} / [1-p^i - (1-p)^i],$$

$$L(i, 0, \underline{k}) = \left\{ \delta + \sum_{\ell=1}^{i-1} \binom{i}{\ell} p^\ell (1-p)^{i-\ell} L(\ell, 0, \underline{i-\ell+k}) \right\} / [1-p^i - (1-p)^i].$$

In the above equations, we use an underline to represent where the station  $m$  with the packet of interest lies among the three classes of stations consisting of those who are competing for transmission, those who had flipped "tail" and are passively monitoring the channel, and those who are in the deferred state. Similar observations and arguments in obtaining the  $C_k$ 's can be applied to the recursive equations of  $L_k = L(\underline{k}, 0, 0)$ . One can show that  $L_k$  is also a continuous function of  $p$  in the open interval  $(0, 1)$ , and  $L_k = +\infty$  when  $p$  is either 0 or 1. Also,  $L_k$  approaches  $+\infty$  as  $p$  approaches 0 or 1. Hence a minimum of  $L_k$ ,  $2 \leq k \leq n$ , exists. Similar to the fact that  $C_k - kC_1$  is independent of  $C_1$ ,  $L_k - \{E[\tau_m] + (k-1)C_1/2\}$  is independent of either  $E[\tau_m]$  or  $C_1$ . Consider the case where  $k$  packets including the packet of interest are transmitted sequentially; then the average time until the packet of interest is transmitted is given by  $E[\tau_m] + (k-1)C_1/2$ . Thus  $L_k$  is always lower bounded by  $E[\tau_m] + (k-1)C_1/2$ . We will call the term  $L_k - \{E[\tau_m] + (k-1)C_1/2\}$  the average collision delay time for station  $m$ . In Fig. 3 we present a plot for the average collision delay time for station  $m$  for various values of  $k$  where we assume that  $E[\tau_m]$  and  $C_1$  are fixed.

We will now investigate the efficiency of the Enet II protocol. As in [1] we will define the efficiency to be the ratio of the average time to transmit a packet without having experienced a collision to the average time to successfully transmit a packet in general. Let  $P_1$  be the probability of a packet being transmitted and experiencing no collision. Let  $P_k$  be the probability that a packet being transmitted experiences a  $k$ -way collision. Then the efficiency  $\mathcal{E}$  is given by

$$\mathcal{E} = \frac{C_1}{C_1 P_1 + \sum_{k=2}^n P_k C_k / k} \quad (6)$$

Notice that we can always lower bound  $\mathcal{E}$  by upper bounding the denominator.

Consider the denominator of (6). It is obvious that maximum efficiency is achieved by minimizing the denominator. Note that in the expression of the denominator, the  $P_k$ 's depend on the packet arrival statistics, and in practice, determination of the  $P_k$ 's is often seen as a challenging problem (It is noted [2] that even if the packet arrivals were assumed to be independent and follow a Poisson arrival distribution, a CSMA/CD protocol can exhibit a packet transmission pattern with little similarity to a Poisson process.) We are now going to find an upper bound for the denominator of (6) by the use of some characteristics of the  $P_k$ 's and  $C_k$ 's. Note that the  $P_k$ 's take values between 0 and 1 regardless of the packet arrival distribution. Hence we have the following inequality for the minimum of the denominator

$$C_1 \leq \min_p \left\{ \sum_{k=1}^n P_k C_k / k \right\} \leq \max_k \left\{ \min_p C_k / k \right\}.$$

Let

$$\mathcal{E}_B = \frac{C_1}{\max_k \left\{ \min_p C_k / k \right\}};$$

then the maximum efficiency  $\mathcal{E}_{\max}$  satisfies the following inequality:

$$\mathcal{E}_B \leq \mathcal{E}_{\max} \leq 1.$$

In Fig. 4, we present plot of  $C_k/k$  as a function of  $p$  for various values of  $k$ . We observe in this case that  $\min_p C_k/k$  increases as  $k$  increases, and as  $k$  gets large,  $\min_p C_k/k$  tends to be close to  $\min_p C_{k+1}/(k+1)$ . We can determine  $\mathcal{E}_B$  numerically by consulting Fig. 4 for this case; that is, if  $n=8$ ,  $C_1=20$ ,  $\delta=2$ , and  $r=1$ , then  $\mathcal{E}_B$  is greater than 83%. However, if we know the prior probability that a packet is transmitted without experiencing a collision, we can further lower bound the maximum efficiency. That is, if  $P_1$  is fixed, then

$$\min_p \left\{ \sum_{k=1}^n P_k C_k / k \right\} = P_1 C_1 + \min_p \left\{ \sum_{k=2}^n P_k C_k / k \right\} \leq P_1 C_1 + (1-P_1) \max_k \left\{ \min_p C_k / k \right\}.$$

In the situation represented by Fig. 4, if we assume that  $P_1=0.5$ , then we have that  $0.908 < \mathcal{E}_{\max} \leq 1$ . Alternatively, if we know a lower bound on  $P_1$  we can repeat a similar argument as above.

Recall that  $C_1^* = C_1$  and that for  $k \geq 2$ ,  $C_k^* \geq C_k$ , where the  $C_k^*$ 's are obtained by upper bounding the  $\mu_j$ 's by  $r/2$ . Let  $P^*$  be a lower bound for  $P_1$ . Then based on the above, we have that  $\mathcal{E}_B^* \leq \mathcal{E}_{\max} \leq 1$ , where

$$\mathcal{E}_B^* = \frac{C_1}{C_1 P^* + (1-P^*) \max_k \left\{ \min_p C_k^* / k \right\}}.$$

Notice that as the average packet transmission time  $C_1$  increases, the lower bound  $\mathcal{E}_B^*$  on the maximum efficiency  $\mathcal{E}_{\max}$  increases. It would be reasonable to anticipate that with longer packets, the time lost to collision resolution becomes small compared to the packet transmission time, and thus the efficiency would increase. For example, with  $P^*=0$ ,  $\delta=2$ ,  $r=1$ , and with  $n=10$ , we have that  $\mathcal{E}_B^* > 68.7\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 81.4\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 89.7\%$

for  $C_1=40$ , and  $\mathcal{E}_B^* > 94.6\%$  for  $C_1=80$ ; for  $n=20$ , we have  $\mathcal{E}_B^* > 66.1\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 79.6\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 88.6\%$  for  $C_1=40$ , and  $\mathcal{E}_B^* > 93.9\%$  for  $C_1=80$ . For  $P^*=0.5$ ,  $\delta=2$ ,  $r=1$ , and with  $n=10$ , we have that  $\mathcal{E}_B^* > 81.4\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 89.7\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 94.6\%$  for  $C_1=40$ , and  $\mathcal{E}_B^* > 97.2\%$  for  $C_1=80$ ; for  $n=20$ , we have  $\mathcal{E}_B^* > 79.6\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 88.6\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 93.9\%$  for  $C_1=40$ , and  $\mathcal{E}_B^* > 96.9\%$  for  $C_1=80$ . Finally, for  $P^*=0.9$ ,  $\delta=2$ ,  $r=1$ , and with  $n=10$ , we have  $\mathcal{E}_B^* > 95.6\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 97.7\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 98.8\%$  for  $C_1=40$ , and  $\mathcal{E}_B^* > 99.4\%$  for  $C_1=80$ ; for  $n=20$ , we have  $\mathcal{E}_B^* > 95.1\%$  for  $C_1=10$ ,  $\mathcal{E}_B^* > 97.5\%$  for  $C_1=20$ ,  $\mathcal{E}_B^* > 98.7\%$  for  $C_1=40$ , and  $\mathcal{E}_B^* > 99.3\%$  for  $C_1=80$ . It might be interesting to compare our results with the results in [2] where a "typical" Ethernet performance is measured and presented.

#### IV. CONCLUSION

In this paper, we presented an analysis of a CSMA/CD collision resolution scheme, namely, the Enet II protocol. We gave recursive expressions for the average collision resolution time and for the average collision delay time of a collision involving  $k$  stations which transmit packets of various packet lengths. We also presented calculations of the lower bounds of the maximum efficiency of the Enet II protocol. We would like to point out that the model we studied is an asynchronous network; and, although we assumed the independence of the packet arrivals at the stations, the analysis is nonparametric in the sense that the result is obtained without assuming any specific packet arrival distribution.

#### ACKNOWLEDGEMENT

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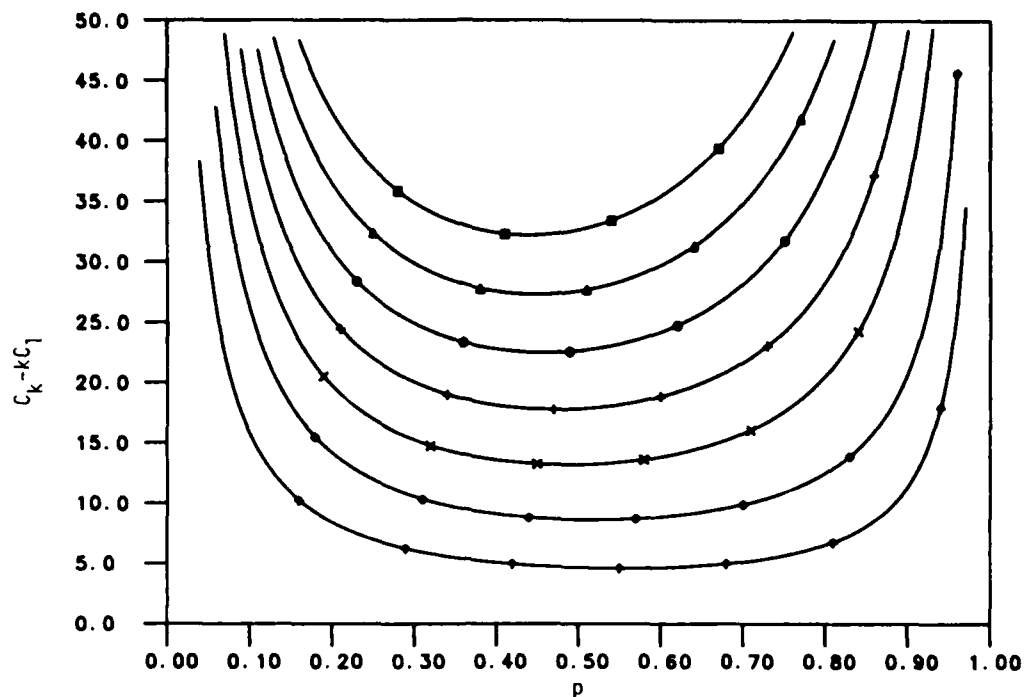


FIG. 1

$\square$  :  $K = 8$   $r = 1$   
 $\triangle$  :  $K = 7$   $\delta = 2$   
 $\circ$  :  $K = 6$   
 $+$  :  $K = 5$   
 $\times$  :  $K = 4$   $\mu_j = (j+2)/[10(j+1)], j \geq 1$   
 $\diamond$  :  $K = 3$   
 $\nabla$  :  $K = 2$

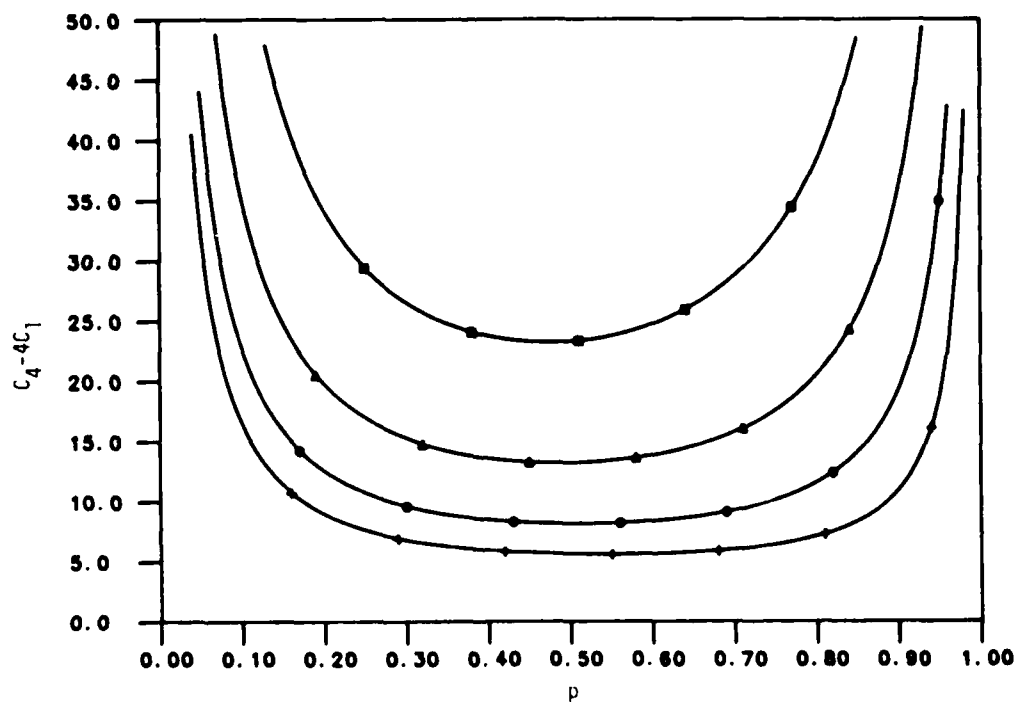


FIG. 2

$\square$  :  $\Delta = 4$   $r = 1$   
 $\triangle$  :  $\Delta = 2$   
 $\circ$  :  $\Delta = 1$   $\mu_j = (j+2)/[10(j+1)], j \geq 1$   
 $+$  :  $\Delta = 0.5$

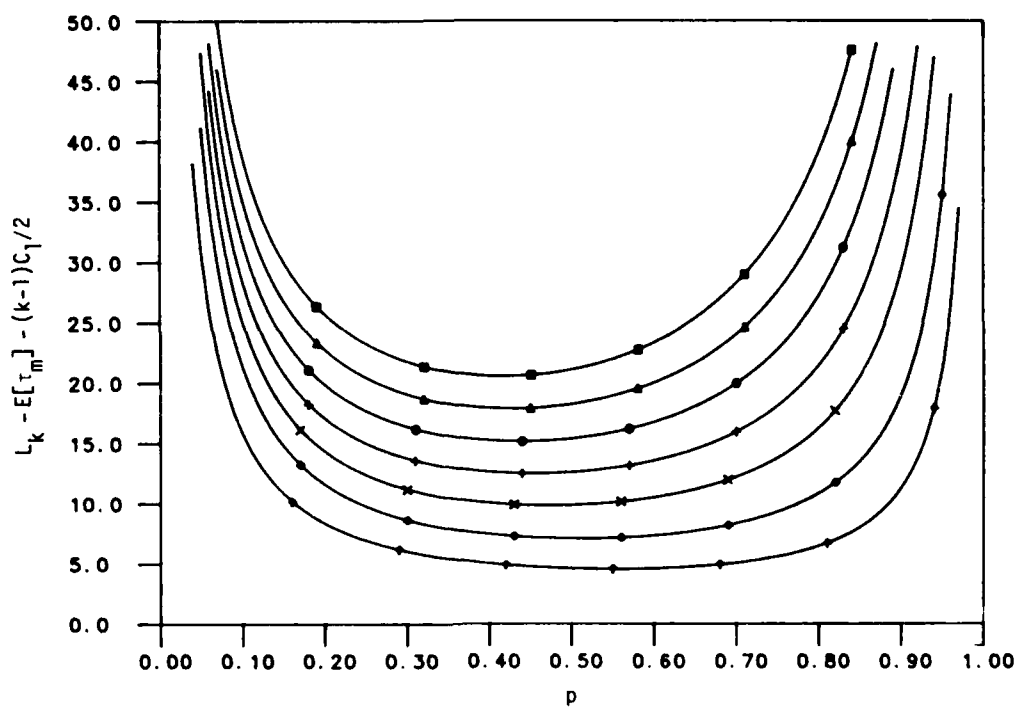


FIG. 3

$\square$  :  $k = 8$   $r = 1$   $E[\tau_m] = 30$   
 $\triangle$  :  $k = 7$   $\delta = 2$   $\mu_j = (j+2)/[10(j+1)], j \geq 1$   
 $\circ$  :  $k = 6$   $C_1 = 20$   
 $+$  :  $k = 5$   
 $\times$  :  $k = 4$   
 $\diamond$  :  $k = 3$   
 $\blacklozenge$  :  $k = 2$

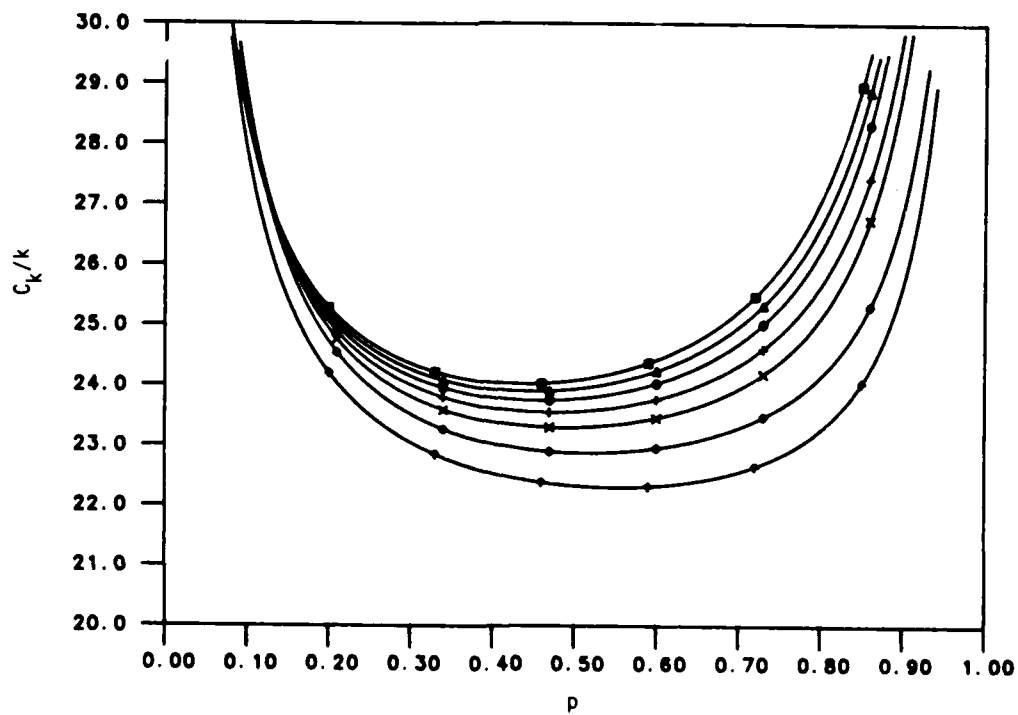


FIG. 4

$\square$  :  $k = 8$   $r = 1$   
 $\triangle$  :  $k = 7$   $\delta = 2$   $\mu_j = (j+2)/[10(j+1)], j \geq 1$   
 $\circ$  :  $k = 6$   $C_1 = 20$   
 $+$  :  $k = 5$   
 $\times$  :  $k = 4$   
 $\diamond$  :  $k = 3$   
 $\blacklozenge$  :  $k = 2$

END

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